Errors in Humphreys' cosmological model

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We wish to point out that both of us are evangelical Christians who not only have graduate-level training in general relativity but actually wrote our PhDs specifically in general relativity and went on to lengthy international research careers in general relativity based at Monash University and at the University of Sydney. We have supervised many students to successful PhDs in general relativity, and some of these former students are now internationally famous in their own right in the relativity research community.

In his 1998 *CEN Tech. J.* paper,[2] 'New vistas of space-time rebut the critics', and in his book,[3] *Starlight and Time: Solving the Puzzle of Distant Starlight in a Young Universe*, D. Russell Humphreys claims to have reconciled the problem of light travel time from distant galaxies with a young-universe cosmology that is based on Einstein's general theory of relativity. Conner and Page[4] on the contrary assert that Humphreys' book and his paper are profoundly flawed and that in fact the cosmological model of *Starlight and Time* is a trivial variant of the standard 'big bang' model, with the definite implication that Humphreys' model actually has the same long time scale as the standard 'big bang' model. The problem then for non-experts in general relativity is how to evaluate the truth of these competing claims regarding a cosmological model within the framework of general relativity.

It is important to note that this problem is concerned precisely with the analysis of claims about general relativity. Now general relativity is a subject which has an unambiguously defined mathematical and physical basis, clearly delineated in classic textbooks such as *Gravitation,*[5] by C.W. Misner, K.S. Thorne and J.A. Wheeler (hereafter cited as MTW), and *Gravitation and Cosmology,*[6] by Steven Weinberg. As such, general relativity is not subject to post-modernist interpretations, and the truth about assertions purporting to be on general relativity can ultimately be unambiguously decided and agreed upon not only by those who are competent in the discipline of general relativity, which means those who are engaged in research and publish in the relativity journals, but also by those with a sufficient level of mathematical competence who take the trouble to work through the details in the classic textbooks. It is our contention that when this is done carefully one finds that Humphreys' book and his paper contain too many physical and mathematical errors to address within the confines of a short paper. We shall therefore restrict our attention in this short paper to many of Humphreys' more serious errors.

One of these is his claim[7] in the *CEN Tech. J.* paper that he has discovered a region of signature change in the Klein metric solution given in his book.

However, as we now show, this claim is false, and in fact is due to his uncritical use of an unphysical coordinate, namely, the Schwarzschild time. In this connection, it is important to note that the resolution of the full structure of the Schwarzschild solution, and in particular the discussion of the physics of the event horizon, requires the *abandonment* of Schwarzschild coordinates and the introduction of either Kruskal-Szekeres coordinates or Novikov coordinates.[8] Schwarzschild coordinates are thus known not to be good global coordinates for the Schwarzschild solution. We now show that they are not good global coordinates for the Klein metric.

The Klein metric that Humphreys uses[9] is given by

$$ds^2 = \beta c^2 dt^2 - \alpha dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right),$$

where

$$\alpha = \frac{1}{1 - \frac{a_m}{a}\eta^2}$$

and

$$\beta = \alpha \frac{\left[1 - \frac{a_m}{a}\left(1 - \frac{\left(1-\eta_e^2\right)^{3/2}}{\left(1-\eta^2\right)^{1/2}}\right)\right]^2}{\left[1 - \frac{a_m}{a}\left(1 - \frac{\left(1-\eta_e^2\right)^{1/2}}{\left(1-\eta^2\right)^{1/2}}\right)\right]^3}$$

It is convenient to use Humphreys' abbreviations, namely, $x = a/a_m$, $\eta = \sin\chi$, $\eta_e = \sin\chi_e$, $\gamma = x - \sin^2\chi$, $\delta = x - 1 + \cos\chi_e / \cos\chi$, and $\varepsilon = x - 1 + \cos^3\chi_e / \cos\chi$.

Then we have

$$\alpha = \frac{x}{\gamma}$$

and

$$\beta = \frac{x^2\varepsilon^2}{\gamma\delta^3}$$

Because it is possible mathematically to have δ either positive or negative while γ is positive, Humphreys asserts that this proves that it is possible to have a region of Euclidean signature in the Klein metric. Unfortunately Humphreys has overlooked a requirement that also has to be satisfied, namely, that all of the coordinate differentials must be real, so that the signature is what is indicated by the metric coefficients.[10] Otherwise one would have to argue that there is a change in signature in going from the real-valued Lorentzian metric of special relativity

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$$

to the 'Euclidean metric'

$$ds^2 = dx^2 + dy^2 + dz^2 + dx_4^2$$

by means of the introduction of the imaginary coordinate $x_4 = ict$.

Obviously for three of the coordinates in Humphreys' Klein metric, namely, r, θ and ϕ, there are no problems about the reality of their differentials. However, the time t, being defined by a rather complicated formula, given incorrectly in Humphreys' book,[11] and not given at all in his *CEN. Tech. J.* paper, is an entirely different matter and needs checking. Note carefully that if t were not given by the complicated formula (given correctly by Conner and Page[12]) the metric would not be a solution of the Einstein field equations. The first part of the check is to look at the behaviour of ζ defined by

$$\zeta = \sqrt{\frac{a_m}{a_m - a}\sqrt{\frac{1-\eta_e^2}{1-\eta^2}} - 1}$$

since t is in fact a function of ζ. Now the above formula for ζ may be written as

$$\zeta = \frac{1}{\sqrt{1-x}}\sqrt{\delta}$$

Since $a < a_m$, $\sqrt{1-x}$ is real, and this means that when $\delta < 0$, ζ is pure imaginary (no real part), i.e. $\zeta = iu$, where u is real. When we substitute this into the correct formula for t we obtain

$$t = \frac{-t_0}{1+b^2}\left[\frac{2ib^3}{1+b^2}\tan^{-1}(u/b) + \frac{iu}{1-u^2} + \frac{1+3b^2}{1+b^2}\left(\frac{\pi}{2} - i\tanh^{-1}u\right)\right]$$

whose varying (u dependent) part is pure imaginary.

The first consequence of this is that dt is pure imaginary in precisely those regions where δ is negative (if it weren't, we would not have a solution of the Einstein field equations in these regions). Hence, instead of being positive, $dt^2 < 0$ in precisely those parts where δ is negative since $i^2 = -1$, and so Lorentzian signature is preserved everywhere (*'minus times minus = plus'*). There is no more signature change involved in the Klein metric than there is in special relativity in using the imaginary coordinate $x^4 = ict$ in order to convert the Lorentzian metric

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$$

to the 'Euclidean metric'

$$ds^2 = dx^2 + dy^2 + dz^2 + dx_4^2.$$

One might have expected this, since a mere change of coordinates cannot alter the signature of the metric. Furthermore, since the lapse function in the Friedmann metric from which the Klein metric is derived has no zero, it is impossible to obtain a change in signature in the Klein metric. Consequently, Humphreys' claim of a signature change in the Klein metric is erroneous, arising because he used a complex-valued unphysical coordinate, coupled with his misunderstanding of the fact that signature is a mathematical concept which has to do with real-valued quadratic forms (the relevant theorem is sometimes known as 'Sylvester's law of inertia'; the mathematical background is fully discussed by Dodson and Poston).[13]

A second major error is his repeated assertion[14] that the full Friedmann-Robertson-Walker (FRW) cosmological model is acentric and that his model, which has an edge and a centre, is fundamentally mathematically and physically different from the FRW model. Both of these statements are incorrect. What Humphreys should have said about centres is that the unbounded FRW solution does not contain a preferred centre. This is very different from saying that it has *no* centre (presumably this is what he means by saying that the unbounded FRW solution is *acentric*). The fact of the matter is that the unbounded FRW solution is such that *any* comoving observer can be taken as the centre of the geometry about which there is perfect spherical symmetry. This means that in the unbounded FRW solution there are an infinite number of possible centres about which there is perfect spherical symmetry. Of course, that means that Humphreys' assertion that the cosmological principle is incompatible with a centre (about which there is spherical symmetry) is incorrect. A careful reading of Section 13.5 and of pages 409–413 of Weinberg's book, and especially the sentence after Weinberg's equation (14.2.7), would have helped to avoid such a mistake.

Conner and Page point out the important fact that the interior solution of Humphreys' model is identical precisely with a portion of a closed FRW universe. The full details of this fundamental fact are spelled out in MTW pages 851–854 in the case of a model collapsing from rest. The same result is obtained by a different route in Weinberg's book, pages 342–345.

Precisely the same result holds true, in the case considered by Humphreys, if the analysis is performed mathematically correctly. Because this is so, it is wrong to argue that models with an edge give rise to large gravitational potentials causing large changes in clock readings. In the case of the collapsing model, MTW states:

> *'Release this star from its initial state, and let it collapse in accord with Einstein's field equations. The interior, truncated Friedmann universe and the*

exterior, truncated Schwarzschild geometry will evolve just as though they had never been cut up and patched together; and this evolution will preserve the smoothness of the match between interior and exterior.'[15]

It is a trivial extension of this result to show that the time and space behaviour of the interior matter region of the cosmological model that Humphreys has attempted to analyse is *identical* to the time and space behaviour of a portion of the Friedmann universe.

Actually, it is worth pointing out at this juncture that neither in his book nor in his *CEN Tech. J.* article does Humphreys anywhere give what general relativists would call a solution with proper mathematical detail. A solution with the required detail would have separate coordinate patches and metrics for (I) the collapsing dust part of the solution, (II) the exterior vacuum solution to this collapsing matter, (III) the expanding dust with nonzero cosmological constant and (IV) the exterior solution with nonzero cosmological constant. As well as this, a proof has to be given that the junction conditions of general relativity are satisfied across the various patches.[16] If Humphreys had done this for Regions I and II he would have discovered that Conner and Page are correct in asserting, in agreement with MTW and Weinberg, that Region I is precisely a truncated part of a full Friedmann dust solution. Humphreys is unable to deal with region III, which in fact needs elliptic functions for its correct solution and matching to region IV. Of greater interest is the fact that the possibility of a valid matching of Region I to Region III is highly problematic.

The only model with correct mathematical details that has been given in these discussions is the one given by Conner and Page. Because of the mathematical identity of Humphreys' interior solution with the interior truncated Friedmann universe and the consequent preservation of the time behaviour of this matter region, Conner and Page are fully justified in entitling their paper *Starlight and Time* is the 'big bang'.

A major error in Humphreys' work, closely connected with his uncritical use of the Schwarzschild time coordinate, is his failure to note that one of the fundamental postulates of general relativity is that the proper time τ_{clock} registered by a clock whose coordinates are given by x^{μ} satisfies the invariant equation

$$c^2 d\tau_{clock}^2 = g_{\mu\nu} dx^{\mu} dx^{\nu},$$

where invariant refers to the fact that the result is independent of the choice of coordinates used to calculate τ_{clock}. For the interior part of the solution where the metric is

$$ds^2 = c^2 d\tau^2 - a^2(\tau)\left(\frac{d\eta^2}{1-\eta^2} + \eta^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right)$$

and for comoving clocks where $d\eta = 0$, $d\theta = 0$ and $d\phi = 0$, we obtain $d\tau_{clock} = d\tau$ which integrates to $\tau_{clock} = \tau$. Since, as we have noted above from MTW[17] and indeed also from Weinberg,[18] the interior solution is a portion of the Friedmann geometry that evolves just as though it had never been truncated from the full Friedmann solution, *the behaviour of comoving clocks in Humphreys' models is exactly the same as the behaviour of comoving clocks in the appropriate portion of the full Friedmann solution*, as Connor and Page had pointed out.

Another error by Humphreys is his assertion[19] that the criterion for an event horizon is $g_{tt} = 0$.

The fact of the matter is that, in non-static solutions of the Einstein field equations, the criterion for an event horizon is not $g_{tt} = 0$. This criterion is sometimes valid for a static geometry such as the Schwarzschild solution (it isn't valid for the maximal analytic extension of the Schwarzschild geometry expressed in Kruskal-Szekeres coordinates), but is definitely not correct otherwise. A very simple counter-example will suffice. According to Humphreys, the horizon is found where $g_{tt} = 0$. Very well, consider the case of the Kerr solution.[20] In Boyer-Lindquist coordinates with $G = 1$ and $c = 1$

$$g_{tt} = \left(1 - 2Mr / \left(r^2 + a^2\cos^2\theta\right)\right).$$

Hence according to Humphreys, the horizon in the Kerr solution should be at $r = M \pm \sqrt{M^2 - a^2\cos^2\theta}$. This is incorrect, as consultation with any standard relativity text will show.[21] $g_{tt} = 0$ gives the so-called static limit, not the event horizon.[22] The outer event horizon occurs at $r = M + \sqrt{M^2 - a^2}$, where in general $g_{tt} \neq 0$. Another example, this time where the metric is diagonal, is the case of collapse of a pressure free homogeneous and isotropic star. The metric appropriate to the interior of the star is that of a section of the Friedmann solution with $k = +1$. For this metric g_{tt} never vanishes, but there is nevertheless a future event horizon, the boundary of the set of outgoing radial null geodesics which pass through the surface of the star before it falls through the Schwarzschild event horizon $R_s = 2GM/c^2$, where M is the mass of the collapsing star. The mathematics of this is analysed in Appendix I of the large paper by Conner and Page (The 'big bang' Cosmology of *Starlight and Time*) and as they point out, this event horizon does not coincide with $g_{tt} = 0$. The real mathematics of event horizons is discussed in Weinberg's textbook pages 490f, and has to do with the convergence or non-convergence of an integral which occurs in the discussion of radial null geodesics.

All of this is not a matter of a 'quibble'; it is a matter of fundamental physics of the light cones on which Humphreys is wrong. The term 'event horizon' has a precise meaning,[23] of which Humphreys seems unaware.

Presumably Humphreys' says that this is all a 'quibble', even if $g_{tt} = 0$ doesn't specify an event horizon in the precise technical language of general relativity, because he claims that large effects occur on clocks when they go through $g_{tt} = 0$. It should be said that quite a number of people made similar errors before the paper by Kruskal in 1960 and the careful analysis of spherically symmetric gravitational collapse *inter alia* by K.S. Thorne and his group at Caltech in the late 1960s. The principal reason for Humphreys making this error is his unfounded belief that the Schwarzschild t coordinate is somehow fundamental. When one looks at the statements in his book, pages 110–113, one sees immediately that Humphreys is unaware of the fact that inside r=2M, the Schwarzschild t direction is no longer timelike but in fact spacelike, so that there is no way that the Schwarzschild t can be used as a time coordinate inside the horizon of the Schwarzschild geometry. MTW states:

> *'Since the spacetime geometry is well behaved at the gravitational radius, the singular behavior there of the Schwarzschild metric components, $g_{tt} = -(1-2M/r)$ and $g_{rr} = (1-2M/r)^{-1}$, must be due to a pathology there of the Schwarzschild coordinates t, r, θ, ϕ. Somehow one must find a way to get rid of that pathology — i.e. one must construct a new coordinate system there from which the pathology is absent. Before doing this, it is helpful to understand better the precise nature of the pathology.*
>
> *The most obvious pathology at r = 2M is the reversal there of the roles of t and r as timelike and spacelike coordinates. In the region r > 2M, the t direction, $\partial/\partial t$, is timelike ($g_{tt} < 0$) and the r direction, $\partial/\partial r$, is spacelike ($g_{rr} > 0$); but in the region r < 2M, $\partial/\partial t$ is spacelike ($g_{tt} > 0$) and $\partial/\partial r$ is timelike ($g_{rr} < 0$).'*[24]

However, Humphreys treats the Schwarzschild t as if it were in the whole of spacetime the reading on a physical clock. But as we stated earlier, the time τ_{clock} measured on an observer's clock is to be calculated from the *invariant* equation

$$c^2 d\tau^2_{clock} = g_{\mu\nu} dx^\mu dx^\nu \, .$$

When one uses this equation and calculates what is observed in the collapsing stage by an observer inside the event horizon, one finds that there is no effect of the like described by Humphreys. No-one should make this kind of error any more because the correct method of analysis has been carefully described in the classic books such as MTW and has also appeared in the texts of a number of other researchers. For example, Norbert Straumann states:

> *'An observer on the surface of the collapsing star will not notice anything peculiar when the horizon is crossed. Locally the space-time geometry is the same as it is elsewhere.'*[25]

A pictorially aided discussion is given on page 848 of MTW. The first diagram on this page also shows clearly the unsuitability of the Schwarzschild coordinate t for the analysis of gravitational collapse.

For these reasons, *based only on the proper analysis of the mathematics and fundamental physics of light-cones and clocks in general relativity*, our conclusion is that Humphreys' attempt to reconcile general relativity with a young-earth viewpoint is flawed. Moreover, Conner and Page present many reasons why a short-age cosmology, based on the assumption that General Relativity holds and on observations of the universe, is impossible. We agree with their mathematics and their result.

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