Creationist cosmologies explain the anomalous acceleration of Pioneer spacecraft

D. Russell Humphreys

A broad class of creationist cosmologies[1] offer an explanation for the 'Pioneer effect', an apparent small Sunward anomalous acceleration of the Pioneer 10 and 11 spacecraft. If a large volume of empty space surrounds the matter of the cosmos, so that the cosmos can have a centre of mass, then the matter is in a deep gravitational potential 'well'. If space is expanding and spreading the matter outward, then the depth of the well is decreasing. According to general relativity, especially a new solution of Einstein's equations derived in the Appendix (which also deals with Birkhoff's theorem), the decreasing depth continuously shortens 'radar' distances within the well, causing the observed apparent acceleration. The magnitude of the anomalous acceleration implies the bottom of the potential well has not yet risen very far above the critical depth for gravitational time dilation. Thus the Pioneer effect supports the essentials of several creationist cosmologies: a centre of mass, expansion of space and recent time dilation.[2] Big bang theorists, whose cosmology does not have a centre of mass, cannot use this explanation. As yet, they have no alternative theory upon which they agree.

The 'Pioneer effect' and its connection with cosmology

In 1998, a team associated with the Jet Propulsion Laboratory, Anderson *et al.*,[3] reported data from the Pioneer 10/11, Galileo and Ulysses spacecraft indicating an 'apparent anomalous, constant, acceleration acting on the spacecraft with a magnitude ~8.5×10^{-8} cm/s^2, directed towards the Sun.' The Pioneer spacecraft are very distant, far beyond the orbit of Pluto. The report prompted a flurry of activity among theorists, who tried to explain the anomaly by using (I) prosaic effects such as gas leaks from the spacecraft,[4–6] (II) exotic new physics,[7,8] or (III) known phenomena previously not applied to the problem.[9] This article is in class III. Anderson *et al.* answered class I comments, making a good case that the cause of the effect is not trivial.[10,11] Then in 2002 they thoroughly documented[12] the effect for Pioneer 10 and 11, the two spacecraft showing the anomaly most clearly. They also surveyed all the theoretical offerings, finding none clearly workable. In late 2005, two of the team's authors surveyed additional evidence and concluded that it supports their original conclusion: the effect is real. 'But', they say, 'neither we nor others with spacecraft or navigational expertise have been able to find a convincing explanation for the anomaly.'[13]

The Pioneer anomaly manifests itself in two ways: (A) as a shortening of the radar (transponded) range relative to the expected one, and (B) as a steady increase of the received (transponded) radio frequency relative to the expected one, i.e. a 'blue shift'. After evaluating systematic and random errors, they list in their eq. (53) their best estimate of the anomalous acceleration: $a_p = -(8.74 \pm 1.33) \times 10^{-8}$ cm/s^2, where the minus sign represents Sunward acceleration. In and around their equation (57), Anderson *et al.* point out (as have many others) that their value for a_p is approximately equal to the Hubble constant H times the speed of light c:

$$a_p \approx -H\,c \qquad (1)$$

If this relationship were exact, then their preferred value for a_p above would imply that the Hubble constant is:

$$H = (90 \pm 14)\,\frac{\text{km/s}}{\text{Mpc}} \qquad (2)$$

This range of values overlaps the currently accepted range of values for the Hubble constant,[14] although the midpoint above is a bit on the high side. This possible connection with cosmology is what aroused the interest of so many theorists. In this article I offer a straightforward explanation for such a connection.

A centre and expansion imply increasing gravitational potential

The usual Friedmann–Lemaître–Robertson–Walker (FLRW, big bang) cosmologies assume that *all* space is filled with an approximately uniform density of matter-energy. As I have pointed out,[15,16] that means such cosmologies do not have a centre of mass within the three-dimensional space we inhabit. To make an alternative cosmology, we could suppose that a large volume of empty space surrounds the matter, in which case the matter would have a centre of mass. The Bible supports such a view.[17] On that basis I suggested a centre-of-mass 'white-hole' cosmology in 1994.[18] In 2003 two non-creationist mathematicians proposed a similar one in the secular literature, though their white hole had no recent time dilation.[19] I know of two other creationist cosmologies with a centre of mass.[20,21]

The matter in a centre-of-mass cosmos would be deep within a 'well' of gravitational potential Φ. Many distributions of matter would give potentials resulting in similar conclusions as those of this article. Figure 1 shows one example. This new model is different than the one in my book because it accounts for the 'waters above the heavens' mentioned in Psalm 148:4 and other Scriptures.[22] As I implied in the book (but did not make clear),[23] the mass of the waters above the heavens could be much greater than the mass of all the stars of the cosmos. In that case, the potential within the shell of waters would be nearly flat[24] compared to the shape of the potential outside the waters.

Neglecting (though we could account for it at the expense of some clarity) the relatively small average curvature of space due to the mass[25] of the galaxies within the 'waters above', the Newtonian gravitational potential would be:[26]

$$\Phi(r) = \begin{cases} -\dfrac{GM}{R}, & \text{for } r \le R \\ -\dfrac{GM}{r}, & \text{for } r > R \end{cases} \tag{3a,b}$$

where r is the radial distance from the centre of mass, R is the radial distance from the centre to the 'waters above', M is their mass, and G is the Newtonian gravitational constant. In my book I reviewed biblical and scientific evidence that the fabric of space has expanded and perhaps is still expanding.[27] As the expansion of space proceeds, the matter (i.e. the distances between galaxies or anything larger) expands with it, so the radius R becomes larger also. Eq. (3a) and the Appendix (in more detail) show that increasing R makes the potential inside R become less negative, as figure 2 illustrates. That is, as matter becomes more spread out, the potential well becomes proportionally shallower.

Differentiating eq. (3a) with respect to proper time τ (in this case, time as recorded by physical clocks at rest with respect to the earth) gives us the rate of increase of Φ in terms of the rate of increase of the matter radius R:

$$\dot{\Phi} = \frac{GM}{R}\frac{\dot{R}}{R}, \quad \text{for } r \le R \tag{4}$$

Here in the main text (but not in the Appendix) the dots represent partial derivatives with respect to proper time. Throughout this article, Φ is a negative number and $\dot{\Phi}$ is a positive number. Inserting eq. (3a) into eq. (4) simplifies the expression for the rate of increase of the potential:

$$\dot{\Phi} = -\Phi\frac{\dot{R}}{R} \tag{5}$$

For an FLRW metric, the fractional rate of change of the scale factor S of the expansion[28] is the Hubble constant H:

$$\frac{\dot{S}}{S} = H \tag{6}$$

In my cosmology, the radius R of the waters above specifies the amount of stretching of the fabric of space, which in turn determines the amount of red-shifting a photon experiences,[29] just as in an FLRW cosmology. So in my cosmology, R is related to the Hubble constant in the same way as S is in the big bang theories:

$$\frac{\dot{R}}{R} = H \tag{7}$$

Using eq. (7) in eq. (5) gives us:

$$\dot{\Phi} = -\Phi H \tag{8}$$

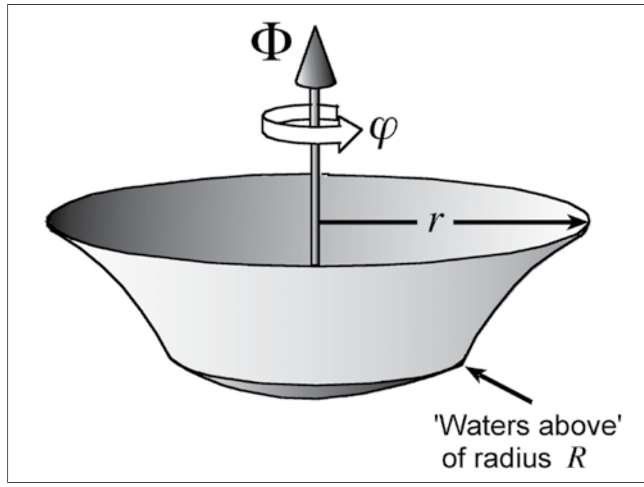

Figure 1. Gravitational potential $\Phi(r, \theta, \varphi)$, with θ suppressed, caused by spherical shell of matter (the 'waters above the expanse') at distance R from the centre.

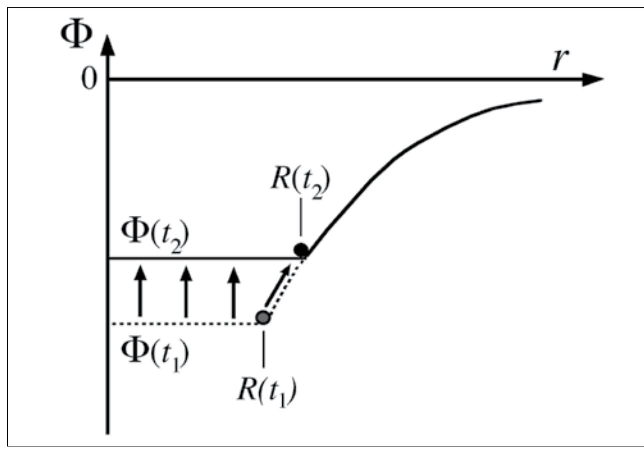

Figure 2. Stretching space increases the radius $R(t)$ of the waters above the heavens, which in turn increases the gravitational potential $\Phi(r, t)$ inside $R(t)$. Time t_2 is greater than time t_1. The potentials inside R are flat, neglecting the small mass (relative to that of the waters above) of all the galaxies within R.

Increasing gravitational potential speeds up radar pulses

Next, let us examine the general-relativistic effects of a changing potential. The Appendix shows that the metric[30] below, containing the potential Φ of eq. (3a), is an *exact* solution (both for large potentials and moving shell) of Einstein's gravitational field equations inside $R(t)$:

$$ds^2 = c^2\left(1+\frac{2\Phi}{c^2}\right)dt^2 - \left(1+\frac{2\Phi}{c^2}\right)^{-1}dr^2 \qquad (9)$$

Here dt^2 and $dr^2 \equiv dx^2 + dy^2 + dz^2$ are the squares of intervals of coordinate time and coordinate distance between two spacetime events near each other. They are intervals as would be measured by a system of *conceptual* (independent of physical effects) clocks and rulers distributed throughout space, all synchronized and calibrated with physical clocks and rulers located where Φ is defined to be zero (in this case, for $r \rightarrow \infty$).[31]

Inside R, as I mentioned above eq. (3a), I am approximating the potential Φ as flat (i.e. neglecting the mass of the stars in the cosmos), everywhere equal to the potential at radius R, so for $r \le R$, $\Phi = \Phi(R)$. Because R depends on time, the potential within R also depends on time, so that $\Phi = \Phi(t)$. One innovation in the Appendix is that it shows that in the cavity, the coefficient of dr^2 in metric (9) can (and must) depend on time, and yet it remains a solution of Einstein's field equations. That is contrary to a common interpretation[32] of a well-known theorem by Birkhoff.[33] The Appendix points out a loophole in that interpretation. Another innovation is that inside the shell the metric is exact (as was previously known for the region outside the shell) even for large depths of the potential well, depths on the same order as $-c^2$. We can re-write this metric in a simpler form:

$$ds^2 = c^2\,d\tau^2 - d\ell^2, \qquad (10)$$

where I define the symbols $d\tau$ and $d\ell$ as:

$$d\tau \equiv \left(1+\frac{2\Phi}{c^2}\right)^{1/2} dt \qquad (11)$$

$$d\ell \equiv \left(1+\frac{2\Phi}{c^2}\right)^{-1/2} dr \qquad (12)$$

Relativists will recognize $d\tau$ and $d\ell$ as intervals of proper time and proper distance. Below, for clarity, I will call the proper distance $d\ell$ the 'radar' distance, because setting $ds = 0$ (see end note[34]) for a positive-going radar pulse in eq. (10) gives us

$$d\ell = c\,d\tau, \qquad (13)$$

just as we would expect for the propagation of an electromagnetic signal.

There is an important difference between coordinate distance and radar distance. Imagine two small particles at rest in a small region of space where the gradient of the potential is zero. The coordinate distance between them is δr and the radar distance is $\delta\ell = \delta r(1 + 2\Phi/c^2)^{-1/2}$, according to eq. (12). If the potential Φ in the region changes suddenly (still keeping the gradient zero), we would expect the coordinate distance δr to remain unchanged for at least a moment, because (a) it is the distance measured by an ideal ruler which is unaffected by the change in potential, and (b) the ideally-measured distance between real particles cannot change instantaneously. In other words, I am interpreting the coordinate distance δr as the distance that would be measured by a set of rulers which are not affected by spacetime. These *conceptual* rulers are outside the fabric of space, not conforming to its curvature and not changing length in accordance with its conditions.

In contrast, according to eq. (12), the radar distance $\delta\ell$ would change suddenly with a sudden change in the potential. We can interpret that change as being caused by a change in the propagation speed $dr/d\tau$ (coordinate distance traveled per unit proper time)[35] of a positive-going radar pulse, which we get by using eq. (11) in eq. (9) and setting $ds = 0$:

$$\frac{dr}{d\tau} = \sqrt{1+\frac{2\Phi}{c^2}}\;c \qquad (14)$$

Because the potential Φ is continually becoming less negative, eq. (14) says that in these coordinates, the propagation speed of the radar pulses is continually increasing.

Faster radar pulses cause the Pioneer effect

As we will see below, this increasing propagation speed for radar pulses causes radar distances to continually decrease with an accelerating rate of decrease. First, from eqs. (13) and (14) we can again get eq. (12), but now we can interpret that equation as giving the radar path length $d\ell$ for an infinitesimal coordinate distance dr a radar pulse travels. Next, integrate eq. (12) from zero to r to get the radar path length ℓ of the pulse when it has travelled a coordinate path length r:

$$\ell = \int_0^r \left(1+\frac{2\Phi}{c^2}\right)^{-1/2} dr' \qquad (15)$$

In the case of a radar range measurement, ℓ and r are the round-trip radar and coordinate path lengths. For simplicity, let us make the integral's upper limit r constant with time, corresponding to an object a fixed coordinate distance ½ r from Earth. (I will relax the requirement for fixed r later.) As the potential Φ increases with time, the integrand above decreases. That implies the radar path length ℓ will decrease during successive pulses. Now differentiate eq. (15) with respect to proper time to get the rate of change of the radar path length:

$$\frac{d\ell}{d\tau} = \int_0^r \frac{\partial}{\partial\tau}\left[\left(1+\frac{2\Phi}{c^2}\right)^{-1/2}\right] dr' \qquad (16)$$

Replace the integration variable above with the following:

$$dr' = \frac{dr'}{d\tau'}\,d\tau' \qquad (17)$$

Carry out the differentiation in eq. (16), then use eq. (14) to replace $dr/d\tau$. That gives us the rate of change of the path length for an object of fixed coordinate distance:

$$\frac{d\ell}{d\tau} = -\frac{1}{c}\int_0^\tau \left(1+\frac{2\Phi}{c^2}\right)^{-1} \dot{\Phi}\,d\tau' \qquad (18)$$

The integral's upper limit τ is the proper time a radar pulse takes to travel a round-trip coordinate path length r, having started at $r = 0$ and $\tau = 0$. Again, take the derivative[36] of eq. (18) with respect to proper time to get the acceleration of the radar path length:

$$\frac{d^2\ell}{d\tau^2} = -\frac{\dot{\Phi}}{c^2 + 2\Phi}\, c \tag{19}$$

The radar range L is one-half the round-trip radar path length: $L = \frac{1}{2}\,\ell$. Using that in the equation above gives us the acceleration of the radar range:

$$\frac{d^2L}{d\tau^2} = -\tfrac{1}{2}\,\frac{\dot{\Phi}}{c^2 + 2\Phi}\, c \tag{20}$$

This means successive radar measurements of an object at a fixed coordinate distance from Earth would give a sequence of ranges L_1, L_2, … that would decrease with the acceleration given above. To relate this result to the Hubble constant, use eq. (8) to substitute for $\dot{\Phi}$ above:

$$\frac{d^2L}{d\tau^2} = \tfrac{1}{2}\,\frac{\Phi}{c^2 + 2\Phi}\, H\, c \tag{21}$$

This equation would be the same as eq. (1), $a_p \approx -Hc$, if the potential Φ has the value

$$\Phi \approx -\tfrac{2}{5}\, c^2 \tag{22}$$

Let us determine Φ more exactly, as well as we can, with the Pioneer effect data. Using the fact that 100 km/s per Megaparsec is $3.2408 \times 10^{-18}\ \mathrm{s}^{-1}$ and the measured value of the acceleration in the text above eq. (1), we can write eq. (1) more accurately as:

$$a_p = -\left(\frac{0.8996 \pm 0.1369}{h}\right) H\, c, \tag{23}$$

where $h \equiv$ (measured value of H) / 100 km/s per Mpc. A team using the Hubble Space Telescope to try to determine H as accurately as possible reports that $h = 0.72 \pm 0.08$.[37] Using that range of values in eq. (23) gives:

$$a_p = -\left(1.25^{+0.40}_{-0.30}\right) H\, c, \tag{24}$$

Next, replacing the left-hand side of eq. (21) with the right-hand side of eq. (24) and solving for Φ gives a numerical result for the depth of the potential in our part of the cosmos:

$$\boxed{\Phi = -\left(0.42 \pm 0.02\right) c^2} \tag{25}$$

So according to these considerations,[38] the gravitational potential in our locality today is between $-0.40\ c^2$ and $-0.44\ c^2$. That is not far above the value $-\frac{1}{2} c^2$, which is the level at which large time dilation[39] occurs in the metric of equation (9). Eq. (3a) says that to get a potential of $-\frac{1}{2} c^2$ for $R = 13.8$ billion light years (not far beyond the distance the Hubble Space Telescope can see galaxies); the mass of the waters above would have to be about 8.8×10^{52} kg.[40] That would be more than 20 times the mass of the all the stars, if the cosmos is that size.[41]

To put it another way, using the potential of eq. (25) in the result of my derivation, eq. (21), gives the observed anomalous acceleration of the Pioneer space probes. This acceleration is toward the earth, but the Pioneer spacecraft are so far away that this would be, at present, indistinguishable from the nominally Sunward acceleration.[42]

You might recall that the derivation above had the spacecraft stationary at a fixed coordinate distance r. That was just for simplicity and does not affect the results. For observers on Earth, the apparent (radar) acceleration and velocity would of course add to or subtract from the effects of the already-modelled coordinate acceleration $\ddot{r}$ and velocity $\dot{r}$ of the spacecraft.

Discussion and conclusions

In the second paragraph of this article, I listed the two ways the Pioneer effect manifests itself experimentally. The first one, (A), the shortening of range according to the decreasing round-trip transit time of transponded radio signals, would be a direct consequence of eq. (21). The steady decrease in round-trip time would also apply, on a smaller time scale, to the time between received radio wave crests. That would explain observation (B), the steady increase in the transponded frequency of the signals.

So an expanding cosmos having a centre of mass provides a simple explanation of the Pioneer effect. If this explanation is correct, the anomalous apparent acceleration of the Pioneer spacecraft would be the first local manifestation we have observed of the expansion of the cosmos, and the first evidence an expansion is occurring in the *present*, not just the past.

In contrast, the big bang theory cannot use this simple explanation, because it does not have a center of mass or a large portion of empty space around its matter. Big bang matter would not exist within a gravitational 'well', and so there would be no change of gravitational potential as expansion proceeds. Without such a change, there would be no change in 'radar' distances and thus no apparent anomalous acceleration of spacecraft. In the absence of an alternative explanation of the Pioneer effect that would be compatible with conventional cosmology, the creationist explanation weighs against the big bang theory.

The magnitude of the Pioneer effect, interpreted according to this paper, would mean the depth of the cosmic potential well is not far above the critical potential at which there are large time dilation effects. Because the potential well has been moving upward through the critical level, large time dilations must have occurred in the relatively recent history of the cosmos. If the earth is located near the centre of mass, it would have emerged from the critical level nearly last of all the matter in the cosmos. According to Scripture and geoscience data, only 6,000 years as measured by Earth's clocks have elapsed since then. Combining that information with the present value of H implies that today the earth would still not be far above the critical level, agreeing in general with the value in eq. (25).

In conclusion, the observed anomalous acceleration of distant spacecraft supports the essentials of several creationist cosmologies—a cosmic centre of mass, expansion of space, and recent gravitational time dilation.

Appendix: Exact metric in an empty expanding shell

This appendix is for readers familiar with general relativity. Imagine the waters above the heavens as a thin expanding hollow shell of total mass M and radius $R(t)$. I want to show that the metric of eq. (9) with the potential of eq. (3a) is exact even for large potentials, that is, potentials with gravitational well depths on the order of $-c^2$, and exact for a time-varying wall radius. Metric (9) is to apply inside the shell, but we will start our search *outside* the shell. In that outer region, Birkhoff's theorem[43] shows that the solution of Einstein's equations is, even for this non-static situation, the static Schwarzschild metric:

$$ds^2 = g_{tt}\,dt^2 + g_{rr}\,dr^2 + g_{\theta\theta}\,d\theta^2 + g_{\varphi\varphi}\,d\varphi^2 \qquad \text{(A1)}$$

where

$$g_{tt} = c^2\left(1 + \frac{2\Phi}{c^2}\right), \qquad \text{(A2)}$$

$$g_{rr} = -\left(1 + \frac{2\Phi}{c^2}\right)^{-1}, \qquad \text{(A3)}$$

$$g_{\theta\theta} = -r^2, \qquad \text{(A4)}$$

$$g_{\varphi\varphi} = -r^2 \sin^2\theta, \qquad \text{(A5)}$$

and Φ is the Newtonian gravitational potential outside a spherically symmetric mass M:

$$\Phi(r) = -\frac{GM}{r}, \quad \text{for } r > R. \qquad \text{(A6)}$$

Here in the Appendix, r is the Schwarzschild radial coordinate measured from the centre of mass, G is the Newtonian gravitational constant and (θ, φ) are the usual spherical coordinate angles (colatitude and azimuth).

Textbook derivations of Birkhoff's theorem show that any metric having spherical symmetry, even a non-diagonal metric or one for an empty cavity, can be cast into the diagonal form of eq. (A1) with the last two coefficients, $g_{\theta\theta}$ and $g_{\varphi\varphi}$, given by eqs. (A4, 5).[44] The first two coefficients, g_{tt} and g_{rr}, remain unspecified, not necessarily having the form of eqs. (A2, 3). The derivations start by allowing g_{tt} and g_{rr} to be functions of t and r, but not of θ and φ (which spherical symmetry would prohibit). From this we look for a solution *inside* the shell.

All the metric coefficients $g_{\mu\nu}$ are subject to a boundary condition that is very important to my argument. They must be *continuous* from just outside the shell all the way through to just inside it.[45,46] Otherwise, spacetime (hence clocks and rulers) would change abruptly from one point to the next. That would be not only contrary to ordinary experience, but also hard to imagine theoretically in the absence of some extraordinary physical cause for it.[47] In this particular case, a tenuous cloud of ice particles only 40 m thick (end note 40) does not seem likely to cause a discontinuity in spacetime. The coefficients of the metric are directly related to the gravitational potential Φ, which must also be continuous through the shell.[48] The change of potential $\Delta\Phi$ from outside to inside is less than the maximum gradient (right outside the shell) of Φ times the thickness ΔR of the shell:

$$|\Delta\Phi| \le \frac{GM}{R^2}\Delta R \qquad \text{(A7)}$$

Just outside the shell the potential is $-GM/R$. Using that in eq. (A7), and taking the absolute value again, shows that the fractional change of potential is less than the fractional change of radius as we go through the shell:

$$\left|\frac{\Delta\Phi}{\Phi}\right| \le \frac{\Delta R}{R} \qquad \text{(A8)}$$

From the data in end note 40, $\Delta R/R$ is very small, on the order of 10^{-25}. That means the potential just inside the shell is almost exactly the same as the potential just outside it. The same is true of the metric coefficients. From Einstein's field equations below, we can show that the absolute values of $\partial g_{tt}/\partial r$ and $\partial g_{rr}/\partial r$ are at a maximum just outside the shell. Applying $\partial/\partial r$ to eqs. (A2, 3), evaluating the results at R, and dividing by the relevant metric coefficients shows that the thickness ΔR limits the fractional change in the coefficients as we go through the shell:

$$\left.\begin{aligned} &\left|\frac{\Delta g_{tt}}{g_{tt}}\right| = \left|\frac{\Delta g_{rr}}{g_{rr}}\right| \le \left|\frac{2\Phi(R)/c^2}{1+2\Phi(R)/c^2}\right|\frac{\Delta R}{R}, \\ &\text{where } \Phi(R) = -\frac{GM}{R} \end{aligned}\right\} \qquad \text{(A9)}$$

Unless $\Phi(R)$ is extremely close to the value $-\tfrac{1}{2}c^2$, close enough to overcome the extremely small value of $\Delta R/R$, $\sim 10^{-25}$, there will be very little change in these two metric coefficients from the outside to the inside of the shell (figure A1). That gives us the result:

$$\boxed{\begin{aligned} (g_{tt})_{\text{inside}} &= (g_{tt})_{\text{outside}}, \qquad \text{(A10)} \\ (g_{rr})_{\text{inside}} &= (g_{rr})_{\text{outside}} \qquad \text{(A11)} \end{aligned}}$$

(The same is true of $g_{\theta\theta}$ and $g_{\varphi\varphi}$, but that is not surprising, because they do not depend on potential.) Using $r = R(t)$ in eq. (A6) and inserting the result into eqs. (A2, 3) shows that the metric coefficients g_{tt} and g_{rr} just *outside* the shell depend on time:

$$(g_{tt})_{\text{outside}} = c^2\left(1 - \frac{2GM}{c^2 R(t)}\right) \qquad \text{(A12)}$$

$$(g_{rr})_{\text{outside}} = -\left(1 - \frac{2GM}{c^2 R(t)}\right)^{-1} \qquad \text{(A13)}$$

Then using the boundary conditions (A10, 11) in these equations (A12, 13) shows that the metric coefficients g_{tt} and g_{rr} just *inside* the shell also depend on time in the same way:

$$\left(g_{tt}\right)_{\text{inside}} = c^2\left(1 - \frac{2GM}{c^2 R(t)}\right), \qquad \text{(A14)}$$

$$\left(g_{rr}\right)_{\text{inside}} = -\left(1 - \frac{2GM}{c^2 R(t)}\right)^{-1} \qquad \text{(A15)}$$

Time dependence occurs not merely just inside the shell, but throughout the cavity. To see this, imagine that the shell is motionless at time t_1, then expands for a while and stops again at some time well before time t_2, remaining motionless thereafter. At times t_1 and t_2 the gradient $\nabla\Phi$ of potential within the cavity is zero,[49] as I mentioned in the main text. So at those two times, the potential and the metric coefficients within the cavity are flat, independent of position, as figure 2 in the main text shows. Now consider what happens to the potential and metric coefficients between the two times, when the shell is expanding. Somehow the potential Φ and the metric coefficients g_{tt} and g_{rr} must change from the state at time t_1 to the state at time t_2. That leads to a conclusion that will shortly become very important.

In an empty expanding shell, the metric must change with time:

$$\boxed{g_{tt} = g_{tt}(t), \quad g_{rr} = g_{rr}(t), \quad \text{for } r \le R(t)} \qquad \text{(A16, 17)}$$

This compares well to the analogous electromagnetic situation, an expanding empty spherical shell of electric charge. First, the electric potential φ in the cavity must change with time to match the changing potential on the expanding sphere, just as in the gravitational field case above. Second, a steadily changing and flat electric potential in a charge-free cavity is a solution of the 4-dimensional version of the Laplace equation, $\Box\varphi = 0$, where $\Box$ is the d'Alembert operator.[50] Now we must see whether or not there is an analogous solution of Einstein's field equations inside an expanding shell of mass. The mixed-tensor version of those equations is:

$$G_\mu^{\ \nu} \equiv R_\mu^{\ \nu} - \tfrac{1}{2}\,\delta_\mu^{\ \nu} R = -\frac{8\pi G}{c^4} T_\mu^{\ \nu} \qquad \text{(A18)}$$

$G_\mu^{\ \nu}$ is the Einstein tensor, $R_\mu^{\ \nu}$ is the Ricci tensor, R is the curvature scalar (for this equation only), G is Newton's gravitational constant, c is the speed of light, and $T_\mu^{\ \nu}$ is the energy-momentum tensor. Many textbooks simplify the algebra considerably by expressing the first two metric coefficients as exponentials of two functions of r and t, here called N and L:

$$g_{tt} = e^{N(r,t)}, \quad g_{rr} = -e^{L(r,t)} \qquad \text{(A19, 20)}$$

(Here in the Appendix, L has a different meaning than in the main text.) Setting $G = c = 1$ for now, and substituting (A19, 20), (A4, 5), and (A1) into the Einstein tensor makes explicit the field equations for this metric:[51–53]

$$G_t^{\ t} = -e^{-L}\left(\frac{L'}{r} - \frac{1}{r^2}\right) - \frac{1}{r^2} = -8\pi T_t^{\ t} \qquad \text{(A21)}$$

$$G_r^{\ r} = -e^{-L}\left(-\frac{N'}{r} - \frac{1}{r^2}\right) - \frac{1}{r^2} = -8\pi T_r^{\ r} \qquad \text{(A22)}$$

$$\left.\begin{aligned} G_\theta^{\ \theta} = G_\varphi^{\ \varphi} &= -e^{-L}\left(\frac{N''}{2} - \frac{L'N'}{4} + \frac{N'^2}{4} + \frac{N'-L'}{2r}\right) \\ &\quad - e^{-N}\left(\frac{\ddot{L}}{2} + \frac{\dot{L}^2}{4} - \frac{\dot{L}\dot{N}}{4}\right) = -8\pi T_\theta^{\ \theta} \\ &\quad = -8\pi T_\varphi^{\ \varphi} \end{aligned}\right\} \text{(A23)}$$

$$G_t^{\ r} = -e^{-L}\,\frac{\dot{L}}{r} = -8\pi T_t^{\ r}, \qquad \text{(A24)}$$

$$G_r^{\ t} = -e^{-N}\,\frac{\dot{L}}{r} = -8\pi T_r^{\ t} \qquad \text{(A25)}$$

Here in the Appendix the dot (or prime) means the partial derivative with respect to coordinate time t (or coordinate radial distance r). Up to now I have closely followed the textbook derivations of Birkhoff's theorem, but here I depart from them. The textbooks *assume* that all components of the energy-momentum tensor $T_\mu^{\ \nu}$ must be zero inside the cavity, even if the shell is expanding. Their basis for that assumption is the lack of obvious sources of gravitational field in the cavity. In particular, looking at eqs. (A24, 25), they set $T_t^{\ r} = T_r^{\ t} = 0$. Because (in general) neither $1/r$, e^{-L} nor e^{-N} are zero, the textbook authors conclude that the remaining factor, $\dot{L}$, must be zero to satisfy the equations. That makes them conclude that L cannot be time-dependent. According to eq. (A20), the time rate of change of L determines that of g_{rr}:

$$\frac{\partial g_{rr}}{\partial t} = -e^{L}\,\dot{L} \qquad \text{(A26)}$$

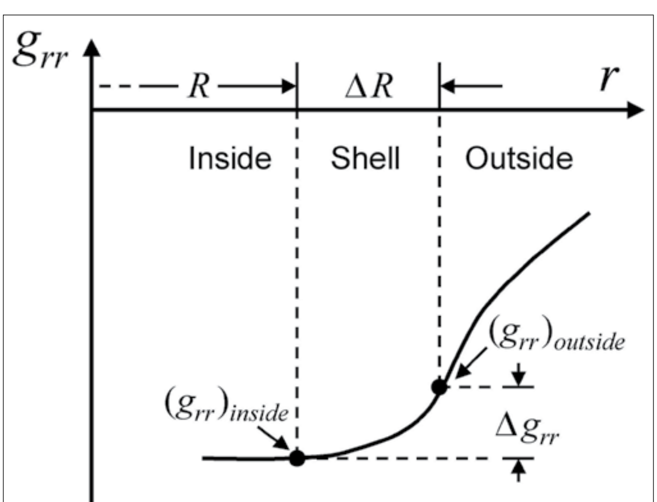

Figure A1. The metric coefficients, such as g_{rr}, must be continuous from outside to inside the shell. Because $\Delta R/R$ is very small, the coefficients are very nearly equal from outside to inside.

So if $\dot{L}$ were zero in the cavity, g_{rr} could not vary with time. That conclusion conflicts with our previous conclusion in eq. (A17). Something must be wrong with the reasoning behind at least one of the two conclusions. Eq. (A17) stems straightforwardly from the continuity of spacetime and seems unassailable. That leads me to question the assumption that all parts of the energy-momentum tensor should be zero, in particular, the assumption that $T_t^{\ r}$ and $T_r^{\ t}$ should be zero. Gravitational potential energy in the cavity is not zero, and in general relativity, all forms of energy—including potential energy —can be sources of gravitational field. So, let us solve eqs. (A24, 25) for $T_t^{\ r}$ and $T_r^{\ t}$ and use eq. (A26) to see what the corresponding components of the energy-momentum tensor (restoring G and c to help interpretation[54]) must be in order to allow g_{rr} to vary with time:

$$T_t^{\ r} = -\frac{c^3 e^{-2L}}{8\pi G r}\frac{\partial g_{rr}}{\partial t}, \qquad \text{(A27)}$$

$$T_r^{\ t} = -\frac{c^3 e^{-(L+N)}}{8\pi G r}\frac{\partial g_{rr}}{\partial t} \qquad \text{(A28)}$$

The possibility of non-zero off-diagonal elements of the energy-momentum tensor in this situation does not appear to have been considered.[55] Later, when we find explicit forms for L, N and g_{rr}, we can simplify the above expressions and try to understand their physical meaning. Non-zero $T_t^{\ r}$ and $T_r^{\ t}$ mean that the usual application of Birkhoff's theorem to a cavity with moving walls has a loophole. The theorem-users' assumption of zero $T_t^{\ r}$ in the cavity does not correspond to reality. So the theorem does not actually exclude the possibility of a time-dependent metric in the cavity.

Moving on toward a solution, it turns out that we can find one by allowing the other terms in the energy-momentum tensor to be zero:

$$T_t^{\ t} = T_r^{\ r} = T_\theta^{\ \theta} = T_\varphi^{\ \varphi} = 0 \qquad \text{(A29)}$$

With their right-hand sides zero, subtracting eq. (A21) from eq. (A22) gives us:

$$N' + L' = 0 \qquad \text{(A30)}$$

As textbooks point out, the solution of that differential equation is

$$N + L = h(t) \qquad \text{(A31)}$$

where h is any function that is independent of r but possibly depending on t. Using eq. (A31) to replace $\dot{N}$ in the second term on the left-hand side of (A23) changes the expression to:

$$\tfrac{1}{2}e^{-N}\left(\ddot{L} + \dot{L}^2 - \tfrac{1}{2}\dot{L}\dot{h}\right) \qquad \text{(A32)}$$

Because $h(t)$ is an arbitrary function, we are perfectly free to choose it such that expression (A32) becomes zero. That is desirable because the two left-hand terms in (A23) would in general be independent of each other. Thus we chose $h(t)$ to be such that:

$$\dot{h}(t) = 2\left(\frac{\ddot{L}}{\dot{L}} + \dot{L}\right), \qquad \text{(A33)}$$

making expression (A32) become zero. The textbooks eliminate the expression in a different way, by requiring L to be independent of time. But eq. (A33) is a perfectly valid alternative for the case that L does depend on time and that its time derivative is not zero. Using eq. (A31) in eq. (A19) modifies the form of the first metric coefficient:

$$g_{tt} = e^{-L+h} \qquad \text{(A34)}$$

However, as the textbooks point out,[56] a simple transformation of the time coordinate,

$$dt'^2 = e^h\, dt^2, \qquad \text{(A35)}$$

will eliminate $h(t)$ from eq. (A34). As the textbooks point out, we can then re-label the time coordinate by dropping the prime from t. All these things make eqs. (A31) and (A19) become:

$$N + L = 0 \qquad \text{(A36)}$$

$$g_{tt} = e^{-L} \qquad \text{(A37)}$$

Using eqs. (A29), (A30), and (A33) in the first three field equations (A21–23) simplifies them:

$$e^{-L}\left(L' - \frac{1}{r}\right) + \frac{1}{r} = 0 \qquad \text{(A38)}$$

$$e^{-L}\left(L' - \frac{1}{r}\right) + \frac{1}{r} = 0 \qquad \text{(A39)}$$

$$L'' - L'\left(L' - \frac{2}{r}\right) = 0 \qquad \text{(A40)}$$

Here I will again depart from the textbooks by pointing out an indefiniteness in the function L as it appears in eqs (A38) and (A39). We can transform L as follows:

$$L = \bar{L} + f(t) \qquad \text{(A41)}$$

where $f(t)$ is an as-yet-unspecified function that can be a constant or depend on time, but cannot depend on r. Then equations (A38) and (A39), which are identical, take the form:

$$e^{-\bar{L}}\left(\bar{L}' - \frac{1}{r}\right) + \frac{e^f}{r} = 0 \qquad \text{(A42)}$$

Similarly to what we did in eq. (A34), using eq. (A41) in eq. (A20) modifies a metric coefficient:

$$g_{rr} = -e^{\bar{L}+f} \qquad \text{(A43)}$$

And like our procedure in eq. (A35), a simple transformation of the radial coordinate,

$$d\bar{r}^2 = e^f\, dr^2, \qquad \text{(A44)}$$

will eliminate $f(t)$ from eq. (A43). This transformation does not affect g_{tt}, but it does change $g_{\theta\theta}$ and $g_{\varphi\varphi}$, as we will see later. Because $f(t)$ does not depend on r, $\bar{L}'$ and L'' in eqs. (A40) and (A42) will not be affected. As before, we can re-label, dropping the bar from L. So the radial metric coefficient again becomes

$$g_{rr} = -e^{L} \tag{A45}$$

This leaves us with two field equations to solve:

$$e^{-L}\left(L' - \frac{1}{r}\right) + \frac{e^{f}}{r} = 0 \tag{A46}$$

$$L'' - L'\left(L' - \frac{2}{r}\right) = 0 \tag{A47}$$

A substitution, $F(r) = e^{-L}$, makes eq. (A46) easy to integrate, giving us:

$$e^{L} = \frac{1}{e^{f} + \dfrac{C}{r}} \tag{A48}$$

where C is an integration constant. Substituting this result into eq. (A47) shows it is a solution of that equation also. From this equation, eq. (A37), and eq. (A46), we find that the first two metric coefficients in the cavity are

$$g_{tt} = e^{f} + \frac{C}{r}, \tag{A49}$$

$$g_{rr} = -\frac{1}{e^{f} + \dfrac{C}{r}} \tag{A50}$$

To match these to the inner boundary conditions at R, equations (A14, 15), we must (restoring c and G) set $C = 0$, and chose $f(t)$ such that:

$$e^{f} = 1 - \frac{2GM}{c^2 R(t)} \tag{A51}$$

Using (A51) and $C = 0$ in (A49, 50) gives us the first two metric coefficients throughout the cavity at all times:

$$g_{tt} = c^2\left(1 + \frac{2\Phi(R)}{c^2}\right), \tag{A52}$$

$$g_{rr} = -\left(1 + \frac{2\Phi(R)}{c^2}\right)^{-1} \tag{A53}$$

where

$$\Phi(R) = -\frac{GM}{R(t)}, \quad \text{for } r \le R(t) \tag{A54}$$

Two well-known relativists agree with the need for a time-dependent g_{tt} in a cavity with moving walls.[57] But because they do not let $T_r^{\ t}$ be non-zero, they do not get a time-dependent g_{rr}. They do not appear to have considered the need for continuity in g_{rr} as well as g_{tt}.

The transformation in eq. (44) affects the remaining two coefficients, $g_{\theta\theta}$ and $g_{\varphi\varphi}$. After few straightforward steps, that equation gives us:

$$r^2 = e^{-f}\,\bar{r}^2 \tag{A55}$$

Using this and eq. (A51) in eqs. (A4, 5) gives us

$$g_{\theta\theta} = -\left(1 + \frac{2\Phi}{c^2}\right)^{-1}\bar{r}^2, \tag{A56}$$

$$g_{\varphi\varphi} = -\left(1 + \frac{2\Phi}{c^2}\right)^{-1}\bar{r}^2\sin^2\theta, \tag{A57}$$

These equations, with eqs. (A52, 53), give us the metric in the cavity:

$$\left.\begin{aligned} ds^2 = c^2\left(1 + \frac{2\Phi}{c^2}\right)dt^2 - \left(1 + \frac{2\Phi}{c^2}\right)^{-1} \\ \left(d\bar{r}^2 + \bar{r}^2\,d\theta^2 + \bar{r}^2\sin^2\theta\,d\varphi^2\right)\end{aligned}\right\} \tag{A58}$$

The rightmost factor is simply the length interval in a 3D flat space. We can express it in Cartesian coordinates x, y and z, so that the metric becomes:

$$\left.\begin{aligned} ds^2 = c^2\left(1 + \frac{2\Phi}{c^2}\right)dt^2 \\ -\left(1 + \frac{2\Phi}{c^2}\right)^{-1}\left(dx^2 + dy^2 + dz^2\right)\end{aligned}\right\} \tag{A59}$$

These coordinates and the isotropic form emphasize that in the flat space of the cavity, no particular origin of coordinates ($r = 0$) is special. Recalling that here

$$\Phi = -\frac{GM}{R(t)}, \quad \text{for } r \le R(t), \tag{A60}$$

we see that this metric is the same as the metric in eq. (9) of the main text of this paper. It is an exact solution of Einstein's equations in a mass-free cavity within an expanding shell of mass.

It remains for us to try to understand the physical meaning of the new, non-zero parts of the energy-momentum tensor, eqs. (A27, 28), that are necessary to get a time-dependent solution. The two parts differ only by a factor $g_{rr}^{\ 2}$ (the covariant forms are equal, $T_{rt} = T_{tr}$), so it is only necessary to examine one of them. Using eqs. (A45) and (A53) in eq. (A27) gives us

$$T_t^{\ r} = \frac{c}{4\pi G r}\,\dot{\Phi}(R) \tag{A61}$$

Multiply numerator and denominator by $r\,\Phi(R)$ to get:

$$T_t^{\ r} = \frac{c\,\Phi(R)}{4\pi G r^2}\, r\,\frac{\dot{\Phi}(R)}{\Phi(R)} \tag{A62}$$

Using eq. (11) of the main text to transform from the coordinate time derivative (the dot here) to the proper time derivative (the dot in the main text) gives:

$$T_t^{\ r} = \sqrt{1 + \frac{2\Phi}{c^2}}\;\frac{c\,\Phi}{4\pi G r^2}\; r\,\frac{\partial\Phi/\partial t}{\Phi} \tag{A63}$$

where here we understand Φ and its time derivative to be evaluated at R. Eq. (8) in the main text shows that $(\partial\Phi / \partial\tau) / \Phi$ is equal to the Hubble constant H. Next, the Hubble constant multiplied by r gives the radial Hubble flow velocity $v(r)$, the velocity (specified using proper time) at radius r of the fabric of space moving away from the centre. Then the right-hand two factors in eq. (A63) become:

$$r\,\frac{\partial\Phi/\partial\tau}{\Phi} = -r\,H = -v(r) \qquad \text{(A64)}$$

Using eqs. (A53) and (A64) in eq. (A63) gives us:

$$T_t^{\ r} = \alpha\,\frac{M\,c}{4\pi\,r^2\,R}\,v(r), \quad \text{where } \alpha \equiv \sqrt{1+\frac{2\Phi}{c^2}} \qquad \text{(A65, 66)}$$

The (r, t) parts of an energy-momentum tensor represent a momentum density (with a factor c making the units the same as those of the diagonal components, energy density).[58] That suggests picturing this equation in terms of momentum flux, a mass per unit area flowing with the Hubble velocity $v\,(r)$ out of the surface of a sphere of radius r and area $4\pi r^2$. (The factor α would become 1 if we were to specify Hubble velocity with coordinate time.) As the fabric of spacetime stretches out, it carries with it the shell's mass-energy Mc^2, which behaves as if it were distributed throughout the cavity by means of the gravitational potential energy.

References

1. By 'creationist cosmology' I mean any cosmology attempting to stay within biblical constraints, such as a young cosmos. My definition does not require that such cosmologies be successful, that is, that they would ultimately prove to meet such requirements.
2. For a creationist cosmology, 'recent' means that, as measured by physical clocks (proper time) on Earth, the cosmos would be approximately 6,000 years old.
3. Anderson, J.D., Laing, P.A., Lau E.U., Liu, A.S., Nieto, M.M. and Turyshev, S.G., Indication from Pioneer 10/11, Galileo, and Ulysses Data, of an apparent anomalous, weak, long-range acceleration, *Physical Review Letters* **81**:2858–2861, 1998.
4. Murphy, E.M., Prosaic explanation for the anomalous accelerations seen in distant spacecraft, *Physical Review Letters* **83**:1890, 1999.
5. Scheffer, L.K., Conventional forces can explain the anomalous acceleration of Pioneer 10, *Physical Review D* **67**:84021–1 to 84021–11, 2003.
6. Katz, J.I., Comment on 'Indication from Pioneer 10/11, Galileo, and Ulysses Data, of an apparent anomalous, weak, long-range acceleration', *Physical Review Letters* **83**:1892, 1999.
7. Bertolami, O. and Páramos, J., The Pioneer anomaly in the context of the braneworld scenario, *Classical and Quantum Gravity* **21**:3309–3321, 2004.
8. Foot, R. and Volkas, R.R., A mirror world explanation for the Pioneer spacecraft anomalies? *Physics Letters B* **517**:13–17, 2001.
9. Whitmore, D.P. and Matese, J. J., The Pioneer 10 anomalous acceleration and Oort cloud comets, *Icarus* **165**:219–222, 2003.
10. Anderson, J.D., Laing, P.A., Lau E.U., Liu, A.S., Nieto, M.M. and Turyshev, S.G., Anderson *et al.* reply, *Physical Review Letters* **83**:1891, 1999.
11. Anderson, J.D., Laing, P.A., Lau E.U., Liu, A.S., Nieto, M.M. and Turyshev, S.G., Anderson *et al.* reply, *Physical Review Letters* **83**:1893, 1999.
12. Anderson, J.D., Laing, P.A., Lau E.U., Liu, A.S., Nieto, M.M. and Turyshev, S.G., Study of the anomalous acceleration of Pioneer 10 and 11, *Physical Review D* **65**:082004-1 to 082004-50, 2002. This fifty-page article answers all the comments I know of, discusses the various theories, and adds much more detail.
13. Nieto, M.M. and Anderson, J.D., Using early data to illuminate the Pioneer anomaly, *Classical and Quantum Gravity* **23**:5343–5354, 2005. Quote from p. 5349. They make several other comments implying that the lack of 'a convincing explanation' includes any as-yet-proposed theories alleging a 'manifestation of unsuspected physics' (p. 5343).
14. Freedman, W.L. *et al*., Final results from the *Hubble Space Telescope* key project to measure the Hubble constant, *Astrophysical Journal* **553**:47–72, 2001. See abstract.
15. Humphreys, D.R., *Starlight and Time*, Master Books, Green Forest, AR, 1994, pp. 96–99.
16. Humphreys, D.R., Our galaxy is the centre of the universe, 'quantized' red shifts show, *Journal of Creation* **16**(2):95–104, 2002. See esp. section 8, 'The big bang can't tolerate a centre,' pp. 100–101; <www.creationontheweb.com/content/view/1570>.
17. Humphreys, ref. 15, pp. 71–72, commenting on the word 'midst' in Genesis 1:6.
18. Humphreys, ref. 15, pp. 100–126.
19. Smoller, J. and Temple, B., Shock-wave cosmology inside a black hole, *Proceedings of the National Academy of Sciences* 100:11216–11218, 2003. They do not put the earth near their centre. In a later interview (Mone, G., Big burp vs. big bang, *Popular Science* **265**(3):154–155, September 2004), Smoller confessed some awareness of previous 'similar ideas' from creation scientists. But, he claimed, 'those theories don't use any mathematics!' He seemed ignorant of the twenty-five equations in the technical appendix of Humphreys, ref. 15, pp. 89–122.
20. Gentry, R.V., *Creation's Tiny Mystery*, 3rd Edition, Earth Science Associates, Knoxville, TN, 1992, pp. 287–290.
21. Hartnett, J., A new cosmology: solution to the starlight travel time problem, *Journal of Creation* **17**(2):98–102, 2003; <www.creationontheweb.com/content/view/1616/>.
22. Humphreys, ref. 15, pp. 58–59, 61–62, 77.
23. Humphreys, ref. 15, p.70. Note phrase 'at least'.
24. Frankel, T., *Gravitational Curvature: An Introduction to Einstein's Theory*, W.H. Freeman and Company, San Francisco, CA, 1979, p. 54. This is a general-relativistic version of Isaac Newton's well-known 'cavity' theorem showing that the empty interior of a stationary spherical shell of mass is free of gravitational force (hence the potential is constant throughout the interior).
25. In a future paper, I hope to add the effect of God creating the matter of the stars in a *time-dependent* way on the 4th day of Creation Week, by creating them in a spherical wave moving out from the earth at the speed of light. That appears to makes the potential increase linearly with distance from the centre out to R, rather than the quadratic increase that a uniform static mass distribution would make. The linear dependence gives a (present, unobserved) age-versus-distance relation that is more desirable than the quadratic potential, especially for nearby stars and neighbouring galaxies.
26. MacMillan, W.D., *The Theory of the Potential*, Dover Publications, New York, 1958, ch. II, sect. 29, p. 40, Table, with $a \rightarrow R$, $b \rightarrow (R - \Delta R)$, $\Delta R << R$, and $4/3\ \pi\ \sigma\ (a^3 - b^3) \rightarrow G\ M$.
27. Humphreys, ref. 15, pp. 66–68, 98.
28. Narlikar, J.V., *Introduction to Cosmology*, 2nd Edition, Cambridge University Press, New York, 1993, ch. 4, p.113, eq. (4.40).
29. More precisely, distance-related redshifts in my cosmology would have *two* causes, not only the stretching of space, but also the change of gravitational potential during the travel time of a photon. Eq. (7) neglects the second effect. It is an approximation that I hope to replace with a more accurate equation in a later paper.
30. Humphreys, ref. 15, p. 89. For those not familiar with general relativity, a metric is a solution of Einstein's gravitational field equations that tells how clocks and measuring rods behave. It specifies the 'interval' *ds* between two nearby points, or 'events', in space-time. In many cases, the interval is the speed of light *c* multiplied by the time registered by a clock moving between the two events.

31. Rindler, W., *Relativity*, Oxford University Press, Oxford, UK, 2001, p.184.

32. Rindler, ref. 31, p. 231.

33. Birkhoff, G.D., *Relativity and Modern Physics*, 2nd Edition, Harvard University Press, Cambridge, MA, pp. 253–256, 1927.

34. For those not familiar with general relativity, setting *ds* equal to zero is a standard way to determine the space-time path of light waves and other electromagnetic waves. It takes advantage of the fact that a physical clock moving along such a path (i.e. at the local speed of light) would not register any time elapsed, making $ds = 0$ (see previous note on metrics).

35. The *coordinate* speed (coordinate distance travelled per unit coordinate time) of radar pulses (and light waves) corresponding to eq. (14) is $dr/dt = (1 + 2\Phi/c^2)\,c$. I interpret potential-induced change in that speed as being caused by the change of energy of the 'fabric of space' brought about by the change of gravitational potential. That is, because the speed of light is the square root of the total energy per unit mass, any change in the total energy density (such as in its gravitational potential component) of the 'fabric' will change the coordinate speed of waves moving through it. That is the underlying reason for the change of 'radar' distance I derive in this paper.

36. Eves, H.; in: Beyer, W.H. (Ed.), *CRC Standard Mathematical Tables*, 24th edition, edited by CRC Press, Cleveland, OH, p. 331, 1976, eq. (49), $q\rightarrow \tau$, $p\rightarrow 0$, $x\rightarrow \tau$.

37. Freedman *et al.*, ref. 14, p. 47.

38. This result depends on eq. (7), which, as I remarked in end note 29, is an approximation. More accurate results await a more accurate accounting of redshifts in a later paper. If the contribution to redshifts from difference of potential is significant, then the potential quoted could be closer to $-\frac{1}{2}c^2$.

39. 'Large time dilation' may well be an understatement, because below the critical potential, $-\frac{1}{2}c^2$, the time-related coefficient of the metric turns negative. For motionless objects, which would have a *dr* of zero, the proper time elapsing would then be *imaginary*, in the mathematical sense. I interpret that to mean that time takes on the characteristics of a spatial dimension. That is, time does not exist below the critical potential. Crudely speaking, time dilation below the critical potential is infinite. Keep in mind that according to eq. (11), there is also a lesser (less than infinite) time dilation that depends on the earth's height above the critical potential.

40. At a distance of 13.8 billion light-years, the thickness of even so great a mass of water would be small. By now it has probably broken up into small particles of ice. If the ice particles have the same average density as the ice particles in Saturn's rings, say about 0.01 kg/m^3 [Allen, C.W., *Astrophysical Quantities*, 3rd Edition, Athalone Press, London, 1973, estimated from data on p. 145], the thickness would be about 40 m.

41. Scott, D., Silk, J, Kolb, E.W. and Turner, M.S., Cosmology; in: Cox, A.N. (Ed.), *Allen's Astrophysical Quantities*, 4th Edition, Springer-Verlag, New York, ch. 26, pp. 654, 649, 2000. The ratio Ω_b of observed 'baryonic' (visible, non-dark, mostly stellar) matter density to the standard FLRW critical density ($1.8788\times10^{-29}\,h^2$ g/cm^3, where $h = H/100$ km/s per Mpc, p. 649) is between $0.007/h^2$ and $0.024/h^2$ (p.654). For stars and galaxies going uniformly all the way out to $R = 13.8 \times 10^9$ light-years, we get a total star mass between 1.2×10^{51} and 4.2×10^{51} kg. The difference of gravitational potential corresponding to that mass of stars between the centre and the 'waters above' would be between 0.0035 c^2 and 0.0119 c^2. That is less than a few percent of the change, 0.5 c^2, from the 'waters above' out to infinity. That justifies my approximation of a flat potential inside *R*.

42. Nieto and Anderson, ref. 13, p. 5352, report a small 'possible annual variation' in the anomalous acceleration of about 0.5×10^{-8} cm/sec^2. It is possible the variation has something to do with the acceleration being along the earth-to-spacecraft line of sight, whose length and direction would vary annually, rather than being in the exact direction of the Sun.

43. Birkhoff, ref. 33.

44. Ohanian, H. and Ruffini, R., *Gravitation and Spacetime*, 2nd Edition, W.W. Norton and Company, New York, p. 398–399, 1994. My derivation follows this derivation closely, using the same sign conventions and symbols.

45. Klein, O., Einige Probleme der allgemeinen Relativitätstheorie, in *Werner Heisenberg und die Physik unserer Zeit*, edited by Bopp, F., Vieweg and Sohn, Braunschweig, pp. 58–72, 1961. English translation available from D. R. Humphreys. In his eq. (76), Klein imposes a continuity condition on the metric coefficients at the boundary between matter and vacuum.

46. Weinberg, S., *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, John Wiley and Sons, New York, p. 346, 1972. Solving the same problem as Klein (above reference) with different terminology, Weinberg also imposes the same continuity condition in his eq. (11.9.37).

47. Hawking, S.W. and Ellis, G.F.R., *The Large Scale Structure of Space-time*, Cambridge University Press, Cambridge, p. 57, 1973.

48. Morse, P.M. and Feshbach H., *Methods of Theoretical Physics*, Part I, McGraw-Hill Book Company, New York, p. 495, 1953. Note 'Dirichlet boundary conditions.'

49. Frankel, ref. 24, p. 54.

50. Pappas, R.C., Proof of 'Birkhoff's theorem' in electrodynamics, *American Journal of Physics* **52**(3):255–256, 1984. Pappas shows from Maxwell's equations that the rate of change, $\dot{\mathbf{E}}$, of the electric field is zero both inside and outside the expanding shell of electric charge. We can use this to deduce potentials. Because **E** outside is not zero, requiring $\dot{\mathbf{E}} = 0$ constrains $\dot{\varphi}$ outside the shell to be zero also. But inside the shell, **E** is zero, so $\dot{\varphi}$ is free to be non-zero. That allows φ to match the time-dependent boundary condition at the shell. Next, the Lorentz gauge condition shows that inside the shell, the magnetic vector potential **A** has a non-zero divergence. The divergence theorem then shows that in the cavity, $\mathbf{A} = -(\dot{\varphi}/3c)\,\mathbf{r}$, where **r** is the radius vector. Thus φ, $\dot{\varphi}$ and **A** are not generally zero inside the shell.

51. Ohanian and Ruffini, ref. 44, p. 399, eqs. [89]–[94].

52. Pathria, R.K., *The Theory of Relativity*, 2nd Edition, Dover Publications, Mineola, New York, p. 215, 1974, eq. (8.19). Note that the sign convention in Pathria's metric and the Einstein field equations differs from that of Ohanian and Ruffini. Otherwise the field equations are the same except for the symbols chosen.

53. Misner, C.W., Thorne, K.S., and Wheeler, J.A., *Gravitation*, W. H. Freeman and Company, New York, p. 844, 1973, eqs. (32.3a–d). These are the covariant form of the equations, and the sign conventions also differ from Ohanian and Ruffini. Otherwise, they are the same.

54. Pathria, ref. 52, p. 215, eq. (8.19), equation for $G_4^{\ 1}$ containing *c*.

55. Pavelle, R., Birkhoff's theorem and Einstein's equations, *Physical Review* **D19**(10):2876–2878, 1979. Pavelle concludes that, 'The necessary and sufficient conditions that Birkhoff's theorem is valid for the Einstein equations $G_j^{\ i} = kT_j^{\ i}$ are that the energy-momentum tensor is static and diagonal.'

56. Ohanian and Ruffini, ref. 44, p. 400, below eq. [98], neglect to square the intervals of time.

57. Bondi, H. and Rindler, W., An addendum to Birkhoff's theorem, *General Relativity and Gravitation* **29**(4):515–517, 1997. The third of their equations (2) appears to lack a $1/r$ factor on the right-hand side. However, they were not deeply concerned about that factor, because they, like every other investigator I know of, assumed that T_{01} must be zero in the cavity.

58. Pathria, ref. 52, p. 99, eq. (3.78).

D. Russell Humphreys, Ph.D., is an Associate Professor of physics for the Institute for Creation Research in California. In 2001 he retired from Sandia National Laboratories in Albuquerque, New Mexico, where he still resides.