

aware of no published retraction of his ‘fossil horse’.

In this letter I outline the serious errors in Humphreys’ reply. The full mathematical critique is available online at creation.com/critical-analysis-in-depth.

Section 1. Continuity across boundaries is not necessary

Humphreys merely reasserts his view that metric coefficients must *necessarily* be continuous. It is true that solutions of differential equations should meet boundary conditions. But solutions also need to satisfy all conditions of the equations, including the presence of surface layers. My solution including a discontinuity at the shell surface layer has the appropriate boundary conditions at all necessary regions: at the origin, at infinity, and at the shell.

The objection to the delta function is irrelevant as my equations (39–40)³ for a cavity inside a layer of mass included no delta function. That solution of the EFE shows that the interior of the cavity is Minkowski space with no ‘timeless zone’.

Another consideration is that the delta function is a concise representation of a thin shell. The delta function captures the integral of the density over the radial distance of a non-zero thickness, i.e.

$$\int \frac{M\delta(r-R)}{4\pi r^2} dV = M = \int_R^{R+\Delta R} \rho(r) dV.$$

The detailed integration of the EFE as in equations (3–5) in Dennis,³ yields for Humphreys’ values $(g_{rr})_{inside}$, $(g_{rr})_{outside}$:

$$\begin{aligned} (g_{rr})_{inside} &= -1 \\ (g_{rr})_{outside} &= -\left(1 - \frac{2M}{r}\right)^{-1}. \end{aligned} \quad (1)$$

In other words, integration over the shell thickness ΔR yields a discontinuity in *inside* and *outside* values, determined by the amount of *gravitational* mass within the shell—a contradiction of Humphreys’ continuity claim. The ‘unrealism’ of the delta function is a red herring.

We return, then, to the *thin layer* representation, to which section 21.13 of MTW⁴ is applicable. In order to support his view of metric coefficients he quotes MTW in the section on surface layers. Unfortunately, his quote indicates that he does not understand the mathematics.

Misunderstanding of MTW and the geometry of surface layers

Humphreys quotes MTW equation (21.169) as support for his claim for the necessity of the continuity of metric components. However, the quote does not support him in the manner in which he enforced continuity. A careful study of section 21.13 of MTW and an understanding of surface layers shows that Humphreys’ version of metric continuity is incorrect. I agree with MTW. MTW does not “directly contradict Dennis”, rather, MTW contradicts Humphreys.

First, the MTW continuity condition concerns the *intrinsic* metric of the three-dimensional hypersurface Σ *containing the mass*, which for the shell is a time-like surface layer. Further, the requirement in equation (21.169) is stated in terms of a *Gaussian normal coordinate system*. In other words, g_{ij} , $i, j = 1, 2, 3$, are the components of the metric tensor within the layer of matter, i.e. in a coordinate system which conforms to the surface layer. This is illustrated in figure 21.6 on page 552 of MTW.⁴ Paying attention to these details is important in understanding surface layers in GR. It may be a surprise that my equation (8):

» Phillip Dennis replies:

I am glad that Dr Humphreys refers to his shell model^{1,2} as a ‘fossil horse’ if that amounts to a public retraction of it, although he recently brought the model to my attention as recently as March 2020, in apparent approbation. I am puzzled that he dug up his fossil horse to apparently present it as a viable model. That was the impetus for my critical analysis. Furthermore, I am

if $r < R(\tau)$:

$$ds^2 = -d\tau^2 + dr^2 + r^2 d\Omega^2$$

if $r > R(\tau)$:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) d\tau^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (2)$$

satisfies equation (21.169) which refers to embedding Σ with *intrinsic geometry* (g_{ij}) in different regions of space-time with different curvatures. The embedding was performed in Appendix A of my critique.³ Clearly, Humphreys did not recognize that Appendix A showed how equation (21.169) is satisfied.

The intrinsic metric of the hypersurface Σ is equation A(45):

$$ds^2 = -d\tau^2 + R^2(\tau)(d\theta^2 + \sin^2(\theta)d\phi^2). \quad (3)$$

The components relevant to equation (21.169) are:

$$g_{ij} = \begin{bmatrix} -1 & & \\ & R^2(\tau) & \\ & & R^2(\tau)\sin^2\theta \end{bmatrix}.$$

Equation (21.169) specifies that the metric of the surface layer is the same when ‘seen from above’ (i.e. as embedded in the Schwarzschild spacetime) and when ‘seen from below’ (i.e. as embedded in the Minkowski spacetime). A summary of Appendix A is that the metrics A(43) and A(44) must

both induce A(45) (equation (3)), i.e. that the shell metric is *isometric* to the boundary of the interior and exterior. Appendix A³ showed that A(45) is continuous across Σ .

Humphreys is wrong since he attempts to enforce continuity (using a spurious appeal to a potential) in Schwarzschild coordinates—which do not conform to the specification of normal coordinates in the hypersurface of matter.

Humphreys says equation (2) above is in error. However, it was obtained by following the method in MTW⁴ (p. 554) for solving the EFE in the presence of surface layers:

“In analyzing surface layers, one uses **not only** the junction conditions (21.168a) to (21.169), **but also** the four-dimensional Einstein field equation applied on each side of the surface Σ separately, **and also** an equation of motion for the surface stress-energy [emphases added].”

I do not deny the continuity at thin layers as explained in MTW. I deny Humphreys’ method. Humphreys’ misinterpretation of MTW is underscored by the fact that after discussing the condition (21.169), MTW gives an exercise to apply the theory. It would be remarkable that MTW would establish the continuity of the *intrinsic geometry* and then turn around and present W. Israel’s⁵ equivalent of my equation (2)

in exercises 21.25–27 in MTW⁴ which Humphreys claims “has a problem”—see figure 1, equations (21.176a, b). If I have a ‘problem’ then so do world-renowned general relativists MTW and W. Israel⁵.

Section 2. My solution appropriately lacks continuity in curvature coordinates

Humphreys takes me to task for presenting a solution with non-continuous coefficients. However, stating this is a ‘problem’, since it runs counter to his thesis is question begging. Per section 1, my equation *does have continuity!* *It is not of the mistaken type that Humphreys claims.*

He then presents a transformation that supposedly reinstates continuity of the metric coefficients for a static shell. He says this transformation makes my solution equivalent to his metric. Humphreys’ external isotropic metric is not a solution of the EFE. Since the exterior is a vacuum, the Ricci scalar R should be zero. However, the computed value from his metric is:⁶

$$R = -\frac{6M^2}{2Mr^3 - r^4}, \quad (4)$$

proving it is not a solution of the EFE. Thus, his putative transform is irrelevant.

A more telling test would be to try to make my (2a) continuous with (2b), both of which are solutions of the vacuum EFE.

Applying Humphreys’ transformations yields:

if $r < R$:

$$ds^2 = -\left(1 - \frac{2M}{R}\right) d\tau^2 + \left(1 - \frac{2M}{R}\right)^{-1} dr^2 + \left(1 - \frac{2M}{R}\right)^{-1} r^2 d\Omega^2$$

if $r > R$:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) d\tau^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Thus, equation (2) does not become the same as his. While this restores continuity to g_{tt} and g_{rr} , it destroys

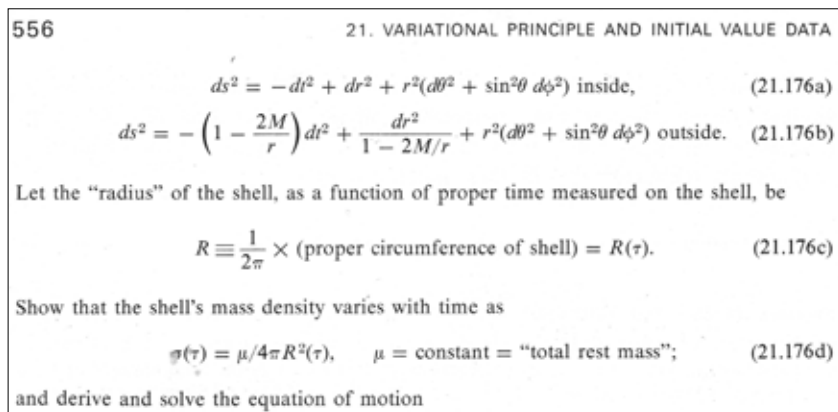


Figure 1. Excerpt from Misner, Thorne, and Wheeler (MTW)⁴

continuity of $g_{\theta\theta}$ and $g_{\phi\phi}$, the very components that are required for continuity of the *intrinsic geometry* of the shell hypersurface as in MTW.⁴ His erroneous transform has led him to further false analysis.

Another indication that his transformation is erroneous is that it changes the metric signature from $(+, -, -, -)$ to $(-, +, +, +)$. This error is the root of his *tri-temporal zone*. *A coordinate transform cannot change the signature of the spacetime metric. This signature change should have raised a red flag.*

To summarize, Humphreys endeavoured to fit my correct interior solution to his erroneous exterior metric. When the attempt is made to fit the interior solution to the correct exterior solution (i.e. eq. (8b)) we find that Humphreys' scale transformation makes the metric discontinuous. Humphreys' claim, "If a changing R makes the transformations above invalid, then he would have to find a different solution for his continuity problem here", is false.

Contra Humphreys, I have no continuity problem and I need no different solution.

Section 3. There is no 'Newtonian potential'—inside a shell it does not change

Humphreys repeats his electromagnetic and potential analogy without validation. Since the analogy leads to equations that are not solutions of the EFE that is a sufficient rebuttal of his 'analogical approach'. Further, Humphreys now admits that a time-dependent potential is not a solution, so how can the 'potential' change?

Discontinuous metric coefficients are not the 'serious error' that he claims. The serious error is Humphreys'. His isotropic metric is not a solution.

The trampoline model refutes Dr Humphreys

After reasserting his potential analogy, he adduces a trampoline to

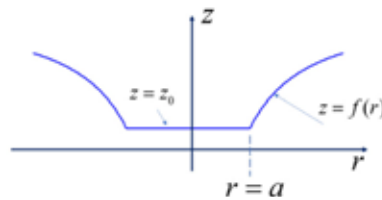


Figure 2. Profile function of a trampoline

illustrate his concepts. He declares this model to *exactly* represent his potential model. He states:

"The depth of the depression (the deviation from the fabric's position with no ring) *corresponds exactly* to the gravitational potential as I am using it in Einstein's equations. This analogy implies the gravitational potential is continuous from outside the spherical shell to the inside, and that the potential inside the shell increases as the radius of the shell increases [emphasis added]."

This is merely a claim; no analytical demonstration is provided. The trampoline surface corresponds to the 'fabric of space', not to a potential. The trampoline model actually refutes Humphreys.

Mathematical model of a trampoline

Figure 2 shows the profile of a trampoline of radius a . The trampoline is generated by rotating the profile $z = f(r)$ about the z -axis. The trampoline surface is embedded in 3D Euclidean space. The metric interval for Euclidean space in cylindrical coordinates is:

$$ds^2 = dr^2 + r^2 d\phi^2 + dz^2 \quad (5)$$

Using standard calculus, the trampoline metric is:

$$\begin{aligned} ds^2 &= \left[1 + (f')^2\right] dr^2 + r^2 d\phi^2 \\ &\equiv g_{rr} dr^2 + g_{\phi\phi} d\phi^2 \end{aligned}$$

$$g_{rr} = \begin{cases} 1 & \text{if } r < a \\ 1 + (f')^2 & \text{if } r \geq a \end{cases}$$

This is manifestly discontinuous for $f' \neq 0$. Thus, contra Humphreys, the trampoline proves that g_{rr} is discontinuous.

Section 4. My solution does not validate Humphreys' solution for the static case

Contra Humphreys I have not validated his isotropic solution for *any* case. *Checking the solution* by substitution is a proper method to validate solutions. If Humphreys had checked his exterior metric, as in equation (4), he would have seen that *his isotropic exterior metric is not a solution*.

I pointed out that Humphreys' exterior metric is not a solution. My equations (18–19)³ are the correct exterior isotropic solution derived in textbooks.

Constant radius is not a solution

The Israel equation of motion⁵ shows that Humphreys' claim of a constant R solution is incorrect. Israel⁵ derived the equation for the acceleration of the shell:

$$\ddot{R} \left[(1 + \dot{R}^2)^{1/2} + \left(1 - \frac{2M}{R} + \dot{R}^2 \right)^{1/2} \right] = -M(1 + \dot{R}^2)/R^2 \quad (6)$$

For constant radius, $R = R_0 > 2M$ and $\dot{R} = 0$, this implies:

$$\ddot{R} = -\frac{M}{R_0^2} \left[1 + \sqrt{1 - \frac{2M}{R_0}} \right]^{-1}.$$

Since \ddot{R} is negative the shell collapses.

Incredibly, Humphreys claims that his violation of tensor transformation laws is not a major mathematical error. He claims this by an appeal to his isotropic form, *which is not a solution*. His equation A(44) with a time-dependent coefficient is most certainly an error. It is obvious that for constant R the function $f(t)$ in equation A(44)

is not a function of time. Then, A(44) would reduce to a valid transformation:

$$d\bar{r}^2 = kdr^2.$$

That is not Humphreys' time-dependent equation A(44), nor would it yield his isotropic metric.

In summary, Humphreys' transformation is a serious mathematical error. That error results in an erroneous metric, inducing a 'tri-temporal' signature change—a 'solution' he still references approvingly in his reply. Committing mathematical errors that result in metrics that do not solve the EFE is a major error no matter how much Humphreys says otherwise.

Section 5. We already have time-dependent solutions

In my critique I presented a time-dependent solution of the EFE with an interior cavity in equation (40)³ which follows from the EFE in comoving coordinates.^{7,8} That equation shows the cavity is Minkowski space with no *timeless zone*.

Additionally, equation (2) is an exact time-dependent solution. The time dependence occurs in the radius $R = R(\tau)$. Equation (2) is the solution presented in exercises 21.25–27 of MTW,⁴ and derived by W. Israel⁵ in his seminal paper.

Section 6. Humphreys' mathematics is flawed and does not connect with the physics

In his concluding section Humphreys claims he has answered the main concerns. This most certainly is not true. Ironically, he says the "mathematics needs to connect with the physics", when his mathematics is flawed and does not connect with physics. First, he has failed to address the issue of the pathological signature change and the 'tri-temporality' of his interior metric—a universe of

one-dimensional beings evolving in three temporal directions. Second, there are no 'timeless zones'.

Conclusions

In summary, Humphreys' errors are:

- Not understanding section 21.13 of MTW. His quote of equation (21.169) as supporting his continuity claim is mistaken.
- Failing to check his isotropic metric.
- Claiming $R = \text{constant}$ is a solution, although the equation-of-motion derived from the EFE shows it is not.
- Adducing another faulty analogy with the trampoline.
- Erroneously asserting without proof that my equation (2) is not a solution.

None of his claims of 'serious errors' in my shell solution are true. His reason is merely that it disagrees with his erroneous potential continuity conjecture.

Finally, Humphreys did not reply to the criticism of his 'timeless zones' and 'tri-temporality'. The 'timeless zones' emerged from his erroneous metric, his misunderstanding of metric signatures and the Schwarzschild ' t ' coordinate. In private communication Humphreys has informed me that he now agrees with the GR community that t for $R < 2M$ is not time. That is tantamount to a retraction of the conclusions of *Starlight and Time* (S&T)⁹ and also of 'a critical potential' that creates 'timeless zones' in his potential model. Rejection of those conclusions invalidates his claims of a solution to the light travel time problem.

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